# Mathematically Modeling the Tastiness of a Pizza

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**Abstract**

Since then, the tastiness of a quintessential pizza is a subjective property. It varies from person to person; some are tasty, some are not. This paper tries to determine and derive the tastiness of a pizza and its components through some possible mathematical models. This paper also aims to "make the perfect-tasting pizza[1]" also by using models. Then the paper determines the feasibility of using these models in real-life applications (i.e. pizza making).

**Keywords:** pizza modeling, models, pizzas, tastiness

## 1 Conventions used in this paper

A component, part, or an ingredient is written in calligraphic font, such as $\mathcal{P}$ and $\mathcal{D}$.

A matrix is written in bolded roman font, such as $\mathbf{A}$.

Other variables used are uppercase and lowercase letters and Greek letters, such as $n$, $T$, and $\alpha$ respectively.

A vector is represented with an italic uppercase and lowercase letters with an over-arrow, such as $\vec{F}$.

A portion of the code of a computer program is written in typewriter (monospaced) font, such as `return True if a % 10 == 3`.

A function is represented with a roman font, such as sin( and gcd(.

## 2 Defining the parts

A typical pizza is made of a pizza dough, sauce (usually made of tomatoes), and toppings of preference of the customer.

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[*] `http://alltootechnical.weebly.com`

[1] That pizza may not be tasty for some people, but deemed perfect-tasting through models.

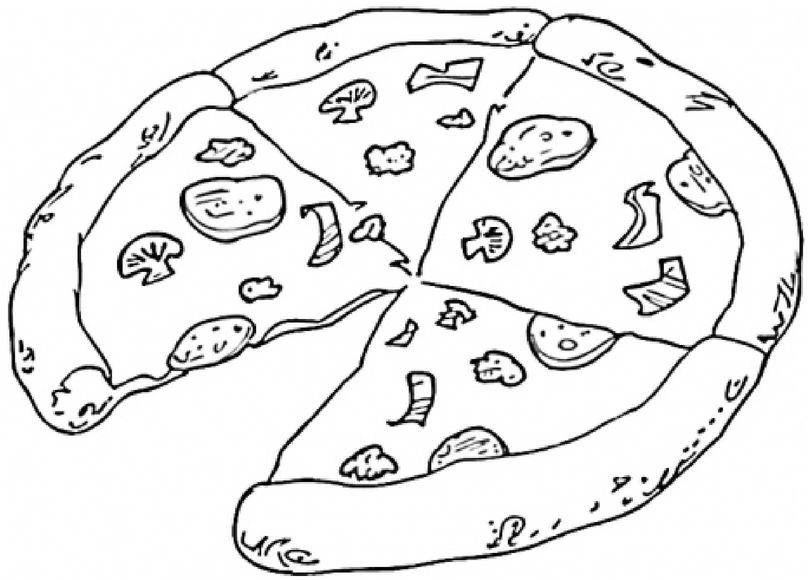


**Figure 0.** *A pizza.*[2]

It is usually divided into slices. The number of slices depend on the radius of the pizza. For instance, a pizza with a radius of 3 inches can be divided into 3 to 4 slices, whereas a pizza with a radius of 30 inches (this is only hypothetical[3]) can be divided into 36 to 42 slices. The optimal number of slices of a pizza with radius $r$ is given by the following equation:

**Proposition 0.**

$$n = \left\{ \left\lfloor \frac{15r}{16} \right\rfloor, \left\lceil \frac{2r}{3} \right\rceil \right\}, \text{where } r \in \mathbb{Z}^+$$

Dividing big pizzas into many slices radially is impractical, as the slices can get absurdly thin. For those cases, a longitudinal division is preferred.

## 2.1 The pizza itself

Suppose we have a pizza $\mathcal{P}$ with radius $r$ and number of slices $n$. A pizza $\mathcal{P}$ is composed of the toppings $\mathcal{T}$, the pizza sauce $\mathcal{S}$, and the dough $\mathcal{D}$. Using these properties, we can define a pizza as:

**Definition 0.**

$$\mathcal{P} = 2\pi r \sum_{k=1}^{n} k\left(\mathcal{T} + \mathcal{D} + \mathcal{S}\right)$$

From Definition 0, we can deduce that there is always pizza sauce $\mathcal{S}$ used in $\mathcal{P}$.

[2]A mere illustration.
[3]Thirty-inch pizzas don't exist yet, but 36-inch pizzas do.

**Postulate 0.**

$$\forall \mathcal{T} \in \mathcal{P}, \quad \exists \mathcal{S} \in \mathcal{P}$$

## 2.2 Pizza sauce

But first, we must define the composition of a pizza sauce $\mathcal{S}$. Most pizza sauces used are always made of tomatoes.[4] The quantity used depends on the surface area and the depth of the whole pizza, taking account the crust in the outermost edge. We can then define the volume of the pizza sauce $V_{\mathcal{S}}$ as:

**Definition 1.**

$$V_{\mathcal{S}} = \frac{1}{3}\pi(r - \text{crust width})a,$$

$$\text{where } a = \text{depth from base to crust height}$$

The quality of the pizza sauce $Q_{\mathcal{S}}$ depends on the ripeness of the tomato $\rho$ and the freshness of the sauce $\sigma$.

**Definition 2.**

$$Q_{\mathcal{S}} = \frac{d}{d\rho}\left(\frac{-\rho^2 + 3\rho}{e^{-\sigma}}\right)$$

Notice that a quadratic function is used for $\rho$. This is because for most people, unripe tomatoes are not tasty, and overripe tomatoes are also not tasty. And the "just right" ripe tomatoes are the tastiest. The aforementioned model conforms to a parabola, hence the quadratic function.

From Definitions 1 and 2, we can derive the composition of the pizza sauce:

**Definition 3.**

$$\mathcal{S} = \frac{V}{2Q}$$

The equation above states that people actually want pizzas with quality sauce rather than loads of sauce with subpar quality[5] (cf. "quality vs. quantity"). The quality of a pizza sauce is an important factor to the tastiness of a pizza.

[4] Some pizzas, however, don't make use of tomatoes as their main ingredient of the sauce.

[5] Subpar quality sauces are often found and manufactured in China.

**Lemma 0.**

$$\text{A pizza } \mathcal{P} \text{ is tasty if } \frac{V_{\mathcal{S}}}{2Q_{\mathcal{S}}} < 0 \quad (Q_{\mathcal{S}} \neq 0).$$

From Definition 3 and Lemma 0, we get:

**Corollary 0.**

$$\text{tastiness}_{\mathcal{P}}(\mathcal{S}) = Q_{\mathcal{S}}^2 \sqrt{V_{\mathcal{S}}}$$

## 2.3 Toppings

Every pizza is unique from one another through its toppings, such as cheese, pepperoni, sausage, bell peppers, garlic, etc. There are infinite ways of putting toppings on a pizza. However, given $t$ number of toppings, there are $t!$ ways to make a pizza. The cheese topping $\mathcal{C}$ will be discussed in a separate section.

There are always limits in how many toppings can you put in the pizza before the toppings $\mathcal{T}$ can easily fall off a slice. Suppose that $\theta$ is the angle of inclination of a pizza slice perpendicular to the pizza box $\mathcal{P}_B$ and the acceleration of the toppings due to gravity $a$. We can model the limit $L$ as:

**Definition 4.**

$$L = \lim_{\mathcal{P}} t = \sup_{n \in \mathcal{P}} \frac{1}{2} a \sin\theta w_t, \quad \forall \mathcal{T},$$

$$\text{where } w_t = \text{total weight of toppings per slice}$$

Too much toppings can be unappetizing for most people. Some people want the pizza simple, with few toppings or none at all except cheese. Since no toppings can be tasty and too much toppings can be not tasty. Therefore:

**Lemma 1.**

$$\text{A pizza } \mathcal{P} \text{ is tasty if } t \to 0, \text{ where } t \in \mathbb{Z}.$$

An exponential model can be correlated from the given statements. A linear model would not be feasible since the differences between $t = 1$ and $t = 2$ vary greatly from $t = 2$ and $t = 3$, and also $t = m$ and $t = m + 1$, where $m \geq 0$.

**Corollary 1.**

$$\text{tastiness}_{\mathcal{P}}(\mathcal{T}) = \frac{1}{e^{t - \frac{7}{6}}}$$

## 2.4 Cheese

Cheese is undoubtedly the most essential of all toppings in a pizza. Some pizzerias offer a 3-cheese, 4-cheese or even a 7-cheese pizza. People like the flavor of the cheese so much that they only want cheese in their pizza. A property of cheeses (especially soft cheeses) that people seem to like the most is their elasticity or "stretchiness" when molten then slightly cooled.

Suppose a pizza $\mathcal{P}$ has a cheese topping $\mathcal{C}$ with elasticity $\epsilon$ and hardness $\eta$. We can define the cheese as:

**Definition 5.**

$$\mathcal{C} = \int_E \mathbf{A} \cdot d\mathbf{r} = \int_{\epsilon_1}^{\epsilon_2} \int_{\eta_1}^{\eta_2} \sqrt{\frac{\epsilon^2}{\ln \eta}}\, d\eta\, d\epsilon,$$

$$\text{where } E = \text{line of the ``string'' formed}$$

The soft cheeses used are usually the tastiest cheeses. People enjoy their pizza whenever they see long strings of cheese that they often eat them. We can deduce that:

**Lemma 2.**

$$\text{A pizza } \mathcal{P} \text{ is tasty if } \epsilon \to \infty$$

From that statement, it implies that:

**Corollary 2.**

$$\epsilon \propto \frac{1}{\eta}$$

As the temperature of the cheese $T$ goes down, the elasticity also goes down. Soft cheeses start to melt at 328 K,[6] while hard cheeses don't melt until 355 K.[7] It implies that temperature and elasticity are directly proportional. A linear model will be suitable for this case. Some cheeses retain their elasticity at room temperature. Therefore:

**Corollary 3.**

$$\text{tastiness}_{\mathcal{P}}(\mathcal{C}) = \begin{cases} \epsilon + k & \text{if } \Delta T > 0 \\ \epsilon & \text{if } \Delta T = 0, \\ \epsilon - k & \text{if } \Delta T < 0 \end{cases}$$

$$\text{where } k = \sqrt{\epsilon \ln T},\ \Delta T = \text{change in temperature}$$

[6] http://en.wikipedia.org/wiki/Cheese
[7] Ibid.

### 2.5 Pizza dough

Another important factor for the tastiness of a pizza $\mathcal{P}$ is the dough used or $\mathcal{D}$. Assume that the pizza dough $\mathcal{D}$ has a radius $r_{\mathcal{D}}$, elasticity $\epsilon_{\mathcal{D}}$, thickness $\tau_{\mathcal{D}}$, and the quality of the dough $Q_{\mathcal{D}}$. Combining these properties to derive the pizza dough $\mathcal{D}$, we get:

**Definition 6.**

$$\mathcal{D} = \pi \sum_{n \in \mathcal{P}} \left(\tau_{\mathcal{D}} \sqrt{r_{\mathcal{D}} \epsilon_{\mathcal{D}}}\right)$$

People like their pizza best when the dough is hand-tossed. When the dough is hand-tossed, it becomes thin, and therefore there are thin-crust pizzas that exist.

**Postulate 1.**

$$\forall\, \tau_{\mathcal{D}} \to \frac{1}{n}, \quad \exists\, \tau_{\text{crust}} \to \frac{1}{n}$$

However, there are special cases of Postulate 1, such as thin pizzas with thick crusts.

**Postulate 2.**

$$\text{If } 0 < \tau_{\mathcal{D}} < \tau_{\text{crust},} \quad \nexists\, \tau_{\text{crust}} \to \tau_{\mathcal{D}}$$

People enjoy thin but crunchy pizzas. And the crunchiness of the pizza has a positive impact of the overall tastiness of a pizza. It implies that the thickness of the pizza $\tau_{\mathcal{P}}$ is inversely proportional to the crunchiness of the pizza $\chi_{\mathcal{P}}$. The cooking temperature $T$ also can affect the crunchiness.

**Lemma 3.**

$$\text{A pizza } \mathcal{P} \text{ is tasty if } \tau_{\mathcal{P}} \to 0 \,\wedge\, \chi_{\mathcal{P}} \to \max_{350 \leq T \leq 450} \frac{1}{\tau_{\mathcal{P}}}.$$

A rather complex but feasible model can be made based from these statements. We get:

**Corollary 4.**

$$\text{tastiness}_{\mathcal{P}}(\mathcal{D}) = \lim_{\mathcal{D} \in \mathcal{P}} \sup_{\epsilon, \tau, \chi \in \mathcal{D}} \left(\frac{\tau_{\mathcal{P}}}{\tau_{\mathcal{D}}} e^{\log_Q \sqrt{\chi_{\mathcal{P}}}}\right)^n$$

### 2.6 Crust

Crusts are the unwanted part of the slice of a pizza for most people. They do not enjoy pizzas with thick crusts. Especially if thick crusts are made on purpose (for instance, the chef rolled the edges of the dough prior to cooking). But, people like crusts when they have the same thickness as the pizza dough used.[8]

More often than not, thick crusts form a torus, the center is the center of the pizza $\mathcal{P}$ itself. Suppose from the pizza dough $\mathcal{D}$, we have the pizza crust $\text{crust}_{\mathcal{D}}$. The volume of $\text{crust}_{\mathcal{D}}$ can be derived from the volume of a torus. The pizza crust $\text{crust}_{\mathcal{D}}$ has the same outer radius as the radius of the pizza $r$, and the inner radius derived from the outer radius as $r - r_{\mathcal{D}}$. Therefore:

**Corollary 5.**

$$V_{\text{crust}_{\mathcal{D}}} = \frac{1}{4}\pi^2 \left(2r - r_{\mathcal{D}}\right) \left(-r_{\mathcal{D}}\right)^2$$

Putting some stuffing inside the crust might be a ludicrous idea, but pizza chains[9] such as Pizza Hut had sold their pizzas with "stuffed crusts." The stuffing was usually sausages. Unfortunately, it didn't appeal much to their customers[10] and eventually phased them out.

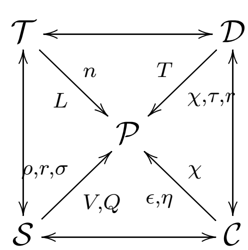
From the above statements, we can deduce that:

**Lemma 4.**

$$\text{A pizza } \mathcal{P} \text{ is tasty if } \nexists\, \text{crust}_{\mathcal{D}} \in \mathcal{P}.$$

## 3 The relation of $\mathcal{P}$ and its parts[11]

Now consider this commutative diagram of the pizza $\mathcal{P}$ and its ingredients (parts):




[8]These thin-crust pizzas are also known as "New York-style" pizzas.
[9]Pizza Hut is not really a pizzeria *per se*.
[10]Apparently, it tasted *really* bad to people.
[11]In the context of this chapter, "parts" are the same as "ingredients".

We can conclude that there exists a bidirectional relationship among $\mathcal{T}$, $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{S}$. Notice that there is only a unidirectional (one-way) relation from $\mathcal{P}$ to $\mathcal{T}$, $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{S}$. That is because if one of the parts are affected, $\mathcal{P}$ is also affected. Meanwhile, the bidirectional relations are because of the interoperability of the parts (Fletcher, et al., 2003).

The commutavity of the parts are sometimes one of the factors of a tasty pizza. But according to Knoll and Nile (1994), the relations between the ingredients of a pizza does not affect the overall tastiness. Therefore, the commutavity of the parts is not a feasible factor in determining the tastiness of the pizza $\mathcal{P}$.

In a general sense, interoperability is defined as the ability of making the parts work together. In our case, the interoperability of the parts is the ability of making the ingredients of the pizza $\mathcal{P}$ “work together” to make $\mathcal{P}$ tasty.

Then we also have the interoperability ratio of the parts, which is described as the ratio of the compatibility of the ingredients to the incompatibility of the ingredients. It was shown that the compatibility is not inversely proportional to the incompatibility, as proven by Oliver (2008).

Suppose we have the interoperability ratio of the parts as $\iota_{\mathcal{P}}$. Then, we define $\iota_{\mathcal{P}}$ as:

**Definition 7.**

$$\iota_{\mathcal{P}} = \frac{\left|\begin{pmatrix} \mathcal{T} & \mathcal{D} \\ \mathcal{S} & \mathcal{C} \end{pmatrix}\right|}{\operatorname{norm}\begin{pmatrix} \mathcal{T} & \mathcal{D} \\ \mathcal{S} & \mathcal{C} \end{pmatrix}},$$

where $\operatorname{norm}(\mathbf{A})$ is the Frobenius norm[12] of the matrix $\mathbf{A}$,

and $|\mathbf{A}|$ is the determinant of $\mathbf{A}$.

Mapping the matrix from Definition 7 to the tastiness functions from Corollaries 0, 1, 3 and 4, we get:

**Corollary 6.**

$$\begin{pmatrix} \mathcal{T} & \mathcal{D} \\ \mathcal{S} & \mathcal{C} \end{pmatrix} \mapsto \begin{pmatrix} \dfrac{1}{e^{t-\frac{7}{6}}} & \lim_{\mathcal{D}\in\mathcal{P}} \sup_{\epsilon,\mathcal{T},\chi\in\mathcal{D}} \left(\frac{\tau_{\mathcal{P}}}{\tau_{\mathcal{D}}} e^{\log_Q \sqrt{\chi_{\mathcal{P}}}}\right)^n \\ Q_{\mathcal{S}}^2\sqrt{V_{\mathcal{S}}} & \epsilon + (\sqrt{\epsilon}\ln T + \Delta T) \end{pmatrix}$$

If $\iota_{\mathcal{P}} \geq 1$, then the ingredients of $\mathcal{P}$ are able to make $\mathcal{P}$ tasty. Therefore:

[12] The Frobenius norm of the matrix $\mathbf{A}$ with size $m \times n$ is the square root of the sum of the squares of each element in $\mathbf{A}$ (i.e. $\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}(\mathbf{A}_{ij})^2}$).

**Lemma 5.**

$$\text{A pizza } \mathcal{P} \text{ is tasty if } \iota_{\mathcal{P}} \geq 1.$$

The ingredients are said to be *marginally* interoperable if $\iota_{\mathcal{P}} = 1$. Meanwhile, ingredients are *truly* interoperable if $\iota_{\mathcal{P}} > 1$. Both are accepted for Lemma 5.

The interoperabilities of $\mathcal{T}$ and $\mathcal{C}$, and $\mathcal{D}$ and $\mathcal{S}$ will be discussed in the next sections.

## 3.1 Interoperability of $\mathcal{T}$ and $\mathcal{C}$

There are no sufficient evidences that there is a bidirectional relation between $\mathcal{C}$ and $\mathcal{T}$. There are two possible ways: $\mathcal{C} \rightarrow \mathcal{T}$ or $\mathcal{C} \leftarrow \mathcal{T}$. We can then express the relation through a mapping:

**Postulate 3.**

$$\mathcal{C} \mapsto \mathcal{T}, \quad \forall n \in \mathcal{P}$$

Even with the absence of $\mathcal{T}$, as long as there is $\mathcal{C}$, a pizza can be tasty. From Postulate 3, we can derive:

**Postulate 4.**

$$\mathcal{T} \not\twoheadrightarrow \mathcal{C}, \quad \forall n \in \mathcal{P}$$

From Postulates 3 and 4, we now have:

**Lemma 6.**

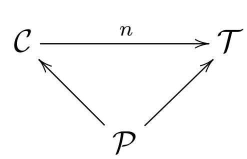
$$\text{A pizza } \mathcal{P} \text{ is tasty if } \mathcal{C} \mapsto \mathcal{T} \wedge \mathcal{T} \not\twoheadrightarrow \mathcal{C}, \quad \forall n \in \mathcal{P}.$$

We can also conclude from Lemma 6 that:

**Corollary 7.**

$$\iota_{\mathcal{P}} \left\langle \; \mathcal{C} \quad \mathcal{T} \; \right\rangle = \text{norm} \left\langle \; \mathcal{C} \quad \mathcal{T} \; \right\rangle^{n}$$

A commutative diagram can be made from the above statements.



## 3.2 Interoperability of $\mathcal{S}$ and $\mathcal{D}$

Similarly from (3.1), there are also insuffcent evidences found for the bidirectional relation between $\mathcal{D}$ and $\mathcal{S}$.

First, we decompose $\mathcal{D}$ and $\mathcal{S}$ into their corresponding parts.

$$\mathcal{S} = \begin{pmatrix} Q_{\mathcal{S}} & V_{\mathcal{S}} & \sigma & \rho \end{pmatrix}$$
$$\mathcal{D} = \begin{pmatrix} \tau & \chi & \epsilon \end{pmatrix}$$

Suppose we represent $[\mathcal{P}]$ as the matrix of its parts:

**Definition 8.**

$$[\mathcal{P}] = \begin{pmatrix} \mathcal{T} & \mathcal{D} \\ \mathcal{S} & \mathcal{C} \end{pmatrix}$$

For this case, we only need $\mathcal{S}$ and $\mathcal{D}$ in $[\mathcal{P}]$. Therefore:

$$\begin{bmatrix} & \mathcal{D} \\ \mathcal{S} & \end{bmatrix} = \operatorname{diag}\left([\mathcal{P}]^{\mathrm{T}}\right) = [\mathcal{P}]^{\mathrm{T}} \cdot \mathbf{I}^{-1},$$
$$\text{where } \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ the } 2 \times 2 \text{ identity matrix}$$

We then factorize $[\mathcal{P}]$ into its lower and upper triangular matrices, $\mathbf{L}$ and $\mathbf{U}$ respectively, using LU decomposition.[13]:

$$[\mathcal{P}] = \mathbf{P}^{-1}\mathbf{L}\mathbf{U},$$
$$\text{where } \mathbf{P} = \text{permutation matrix}$$

Using the general definition of the upper and lower triangular matrices, we now have:

$$[\mathcal{P}] = \mathbf{P}^{-1} \begin{pmatrix} \mathcal{T}_L & 0 \\ \mathcal{S}_L & \mathcal{C}_L \end{pmatrix} \begin{pmatrix} \mathcal{T}_U & \mathcal{D}_U \\ 0 & \mathcal{C}_U \end{pmatrix}$$

Since $\mathbf{I} = \mathbf{I}^{-1}$, we substitute $\mathbf{P}$ for $\mathbf{I}$:

$$[\mathcal{P}] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{T}_L & 0 \\ \mathcal{S}_L & \mathcal{C}_L \end{pmatrix} \begin{pmatrix} \mathcal{T}_U & \mathcal{D}_U \\ 0 & \mathcal{C}_U \end{pmatrix}$$

$[\mathcal{P}]$ can then be simplified into:

$$[\mathcal{P}] = \begin{pmatrix} 0 & \mathcal{D}_U \\ \mathcal{S}_L & 0 \end{pmatrix} = \begin{pmatrix} 0 & \mathcal{D} \\ \mathcal{S} & 0 \end{pmatrix} = \begin{bmatrix} & \mathcal{D} \\ \mathcal{S} & \end{bmatrix}$$

As you can see, $[\mathcal{P}] = \begin{bmatrix} & \mathcal{D} \\ \mathcal{S} & \end{bmatrix}$. It also infers that $\iota_{\mathcal{P}}\left([\mathcal{P}], \begin{bmatrix} & \mathcal{D} \\ \mathcal{S} & \end{bmatrix}\right) = 1$. We could say that $\mathcal{P}$ and $\mathcal{S}$, $\mathcal{D}$ are interoperable. If $\mathcal{P}$ and $\mathcal{S}$, $\mathcal{D}$ are interoperable, it follows that:

[13] $[\mathcal{P}]$ then will be expressed as factors of $\mathbf{L}$ and $\mathbf{U}$.

**Lemma 7.**

$$\text{A pizza } \mathcal{P} \text{ is tasty if } \iota_{\mathcal{P}}(\mathcal{S}, \mathcal{D}) \geqq 1.$$

The inequality only holds for special cases of $[\mathcal{P}] = \begin{bmatrix} & \mathcal{D} \\ \mathcal{S} & \end{bmatrix}$.

# 4 The inequality of the parts

The ingredients of $\mathcal{P}$ vary greatly in proportion. We can then arrange the parts by increasing volume:

$$\mathcal{C} \leq \mathcal{T} < \mathcal{S} < \mathcal{D}$$

The equality holds if $\nexists \mathcal{T} \in \mathcal{P}$.