

You're given a task to build a circular theater  $\mathcal{C}$  with the space available in the parking lot  $\mathcal{P}$ .  $\mathcal{P}$  has a side length of  $l_{\mathcal{P}}$ .

The area of  $\mathcal{P}$  is  $A_{\mathcal{P}} = l_{\mathcal{P}}^2\pi$ . The area of  $\mathcal{C}$  is  $A_{\mathcal{C}} = \frac{l_{\mathcal{P}}^2\pi}{4}$ . The area of the stage in the theater  $\mathcal{S}$  is  $A_{\mathcal{S}} = r_{\mathcal{S}}^2\pi$ , where  $r_{\mathcal{S}}$  is the radius of the stage.

From the given, the area for the available seats can be determined by:

$$A_{\mathcal{C}} - A_{\mathcal{S}} = \frac{l_{\mathcal{P}}^2\pi}{4} - r_{\mathcal{S}}^2\pi$$

In each row of the circular theater, there exists a seat with dimensions  $l_{\text{seat}}$  by  $w_{\text{seat}}$ .

Now, consider the area function:

$$\mathcal{A}(n) = \pi(r_{\mathcal{S}} + w_{\text{seat}}n)^2 - \pi(r_{\mathcal{S}} + w_{\text{seat}} \cdot (n - 1))^2$$

The cumulative area function can be derived from there:

$$\mathcal{A}_{\text{cul}}(n) = \sum_{i=1}^n \mathcal{A}(i) = \pi(r_{\mathcal{S}} + w_{\text{seat}}n)^2 - \pi r_{\mathcal{S}}^2$$

From there, the number of seats given the  $n$ -th row is:

$$\#_{\text{seat}}(n) = \frac{\mathcal{A}(n) - 4 \cdot A_{\text{aisle}}(n)}{l_{\text{seat}}w_{\text{seat}}}$$

The area of the aisle must be taken into account.

$$\begin{aligned} A_{\text{aisle}}(n) &= \int_{-\frac{w_{\text{aisle}}}{2}}^{\frac{w_{\text{aisle}}}{2}} \left| \left[ \sqrt{(r_{\mathcal{S}} + w_{\text{seat}}n)^2 - x} \right] - \left[ \sqrt{(r_{\mathcal{S}} + w_{\text{seat}} \cdot (n - 1))^2 - x} \right] \right| dx \\ &= \frac{\sqrt{2}}{6} \cdot \left[ w_{\text{aisle}} (-\sqrt{f} - \sqrt{g} + \sqrt{p} + \sqrt{q}) + 4r_{\mathcal{S}}w_{\text{seat}} (n (\sqrt{f} - \sqrt{g} - \sqrt{p} + \sqrt{q}) - \sqrt{f} + \sqrt{g}) + \right. \\ &\quad \left. 2w_{\text{seat}}^2 (n (n (\sqrt{f} - \sqrt{g} - \sqrt{p} + \sqrt{q}) - 2\sqrt{f} + 2\sqrt{g}) + \sqrt{f} - \sqrt{g}) + 2r_{\mathcal{S}}^2 (\sqrt{f} - \sqrt{g} - \sqrt{p} + \sqrt{q}) \right] \end{aligned}$$

where,

$$\begin{aligned} f &= 2(r_{\mathcal{S}} + w_{\text{seat}} \cdot (n - 1))^2 - w_{\text{aisle}} \\ g &= 2(r_{\mathcal{S}} + w_{\text{seat}} \cdot (n - 1))^2 + w_{\text{aisle}} \\ p &= 2(r_{\mathcal{S}} + w_{\text{seat}}n)^2 - w_{\text{aisle}} \\ q &= 2(r_{\mathcal{S}} + w_{\text{seat}}n)^2 + w_{\text{aisle}} \end{aligned}$$

A function of  $n$  is thereby obtained.