Identities

Reciprocal

$$\csc\theta=\frac{1}{\sin\theta}$$

$$\sec\theta=\frac{1}{\cos\theta}$$

$$\cot\theta=\frac{1}{\tan\theta}$$

Ratio

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\cot\theta=\frac{\cos\theta}{\sin\theta}$$

Pythagorean

$$\sin^2\theta+\cos^2\theta=1$$

$$\tan^2\theta+1=\sec^2\theta$$

$$1+\cot^2\theta=\csc^2\theta$$

Alternate Forms

$$\sin\theta=\frac{1}{\csc\theta}$$
$$\sin\theta\csc\theta=1$$
$$\cos\theta=\frac{1}{\sec\theta}$$
$$\cos\theta\sec\theta=1$$
$$\tan\theta=\frac{1}{\cot\theta}$$
$$\tan\theta\cot\theta=1$$

$$\sin\theta=\cos\theta\tan\theta$$
$$\cos\theta=\frac{\sin\theta}{\tan\theta}$$
$$\cos\theta=\sin\theta\cot\theta$$
$$\sin\theta=\frac{\cos\theta}{\cot\theta}$$

$$\sin^2\theta=1-\cos^2\theta$$
$$\cos^2\theta=1-\sin^2\theta$$
$$\tan^2\theta=\sec^2\theta-1$$
$$\sec^2\theta-\tan^2\theta=1$$
$$\cot^2\theta=\csc^2\theta-1$$
$$\csc^2\theta-\cot^2\theta=1$$

Sum and Difference

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\alpha\mp\sin\alpha\sin\beta$$

$$\tan(\alpha\pm\beta)=\frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$$

$$\cot(\alpha\pm\beta)=\frac{\cot\alpha\cot\beta\mp1}{\cot\beta\pm\cot\alpha}$$

Double Angle

$$\sin2\alpha=2\sin\alpha\cos\alpha$$

$$\cos2\alpha=\cos^2\alpha-\sin^2\alpha$$
$$=1-2\sin^2\alpha$$
$$=2\cos^2\alpha-1$$

$$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

Power Reduction

(for degree 2)

$$\sin^2\theta=\frac{1-\cos2\theta}{2}$$

$$\cos^2\theta=\frac{1+\cos2\theta}{2}$$

$$\tan^2\theta=\frac{1-\cos2\theta}{1+\cos2\theta}$$

Half Angle

Formula	$\mathrm{sgn}\left(\frac{\alpha}{2}\right)$	Q
$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$	+	I, II
	−	III, IV
$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$	+	I, IV
	−	II, III
$\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$	+	I, III
	−	II, IV

$$=\frac{\sin\alpha}{1+\cos\alpha}$$
$$=\frac{1-\cos\alpha}{\sin\alpha}$$
$$=\csc\alpha-\cot\alpha$$

Multiple Angle

$$\sin3\alpha=3\sin\alpha-4\sin^3\alpha$$

$$\cos3\alpha=4\cos^3\alpha-3\cos\alpha$$

$$\tan3\alpha=\frac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}$$

$$\sin4\alpha=4\sin\alpha\cos\alpha-8\sin^3\alpha\cos\alpha$$

$$\cos4\alpha=8\cos^4\alpha-8\cos^2\alpha+1$$

$$\tan4\alpha=\frac{4\tan\alpha-4\tan^3\alpha}{1-6\tan^2\alpha+\tan^4\alpha}$$

$$\sin5\alpha=5\sin\alpha-20\sin^3\alpha+16\sin^5\alpha$$

$$\cos5\alpha=16\cos^5\alpha-20\cos^3\alpha+5\cos\alpha$$

$$\tan5\alpha=\frac{\tan^5\alpha-10\tan^3\alpha+5\tan\alpha}{1-10\tan^2\alpha+5\tan^4\alpha}$$

Powers

(for degree 3+)

$$\sin^3\theta=\frac{3\sin\theta-\sin3\theta}{4}$$

$$\cos^3\theta=\frac{3\cos\theta+\cos3\theta}{4}$$

$$\sin^4\theta=\frac{3-4\cos2\theta+\cos4\theta}{8}$$

$$\cos^4\theta=\frac{3+4\cos2\theta+\cos4\theta}{8}$$

$$\sin^5\theta=\frac{10\sin\theta-5\sin3\theta+\sin5\theta}{16}$$

$$\cos^5\theta=\frac{10\cos\theta+5\cos3\theta+\cos5\theta}{16}$$

Product-to-Sum

$$\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta=\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$$

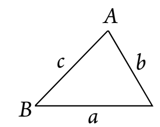
Sum-to-Product

$$\sin\theta\pm\sin\varphi=2\sin\frac{\theta\pm\varphi}{2}\cos\frac{\theta\mp\varphi}{2}$$

$$\cos\theta\pm\cos\varphi=\pm2\cos\frac{\theta+\varphi}{2}\cos\frac{\theta-\varphi}{2}$$

Law of Sines

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$



Law of Cosines

$$a^2=b^2+c^2-2bc\cos A \qquad \cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$b^2=a^2+c^2-2ac\cos B \qquad \cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$c^2=a^2+b^2-2ab\cos C \qquad \cos C=\frac{a^2+b^2-c^2}{2ab}$$

Law of Tangents

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}$$

$$\frac{c-a}{c+a}=\frac{\tan\frac{1}{2}(C-A)}{\tan\frac{1}{2}(C+A)}$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(B-C)}{\tan\frac{1}{2}(B+C)}$$

Trigonometry Formulas
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